SQP SOLUTIONS

21. We have
$$x = \frac{1-t^2}{1+t^2}$$
 and $y = \frac{2t}{1+t^2}$

Putting $t = \tan\theta$ in both the equations, we get

$$x = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$$

and
$$y = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$$
 ...(ii)

Differentiating (i) and (ii), we get

$$\frac{dx}{d\theta} = -2\sin 2\theta$$
 and $\frac{dy}{d\theta} = 2\cos 2\theta$

Therefore,
$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$$

22. (a) Let the direction cosines of the line be l, m, n. Then, $l = \cos 90^\circ = 0$

$$m = \cos 60^{\circ} = \frac{1}{2}$$
 and $n = \cos 30^{\circ} = \frac{\sqrt{3}}{2}$.

So, direction cosines are $<0, \frac{1}{2}, \frac{\sqrt{3}}{2}>$.

OI

22. (b) Here, P(2, 1, 0) and Q(1, -2, 3)

So,
$$PQ = \sqrt{(1-2)^2 + (-2-1)^2 + (3-0)^2}$$

= $\sqrt{1+9+9} = \sqrt{19}$

Thus, the direction cosines of the line joining two points

$$\text{are} < \frac{1-2}{\sqrt{19}}, \frac{-2-1}{\sqrt{19}}, \frac{3-0}{\sqrt{19}} > i.e., < \frac{-1}{\sqrt{19}}, \frac{-3}{\sqrt{19}}, \frac{3}{\sqrt{19}} >$$

23. We have,
$$y = cx + \frac{a}{c}$$
 ...(i)

Differentiating with respect to x, we get $\frac{dy}{dx} = c$

$$\therefore x \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}} = xc + \frac{a}{c} \qquad \qquad \left[\text{Putting } \frac{dy}{dx} = c \right]$$

$$\Rightarrow x \frac{dy}{dx} + \frac{a}{\frac{dy}{dx}} = y$$
 [Using (i)]

Hence, $y = cx + \frac{a}{c}$ is a solution of the differential equation $x\frac{dy}{dx} + \frac{c}{dy}\frac{a}{dx} = y$.

24. (a) We have, L.H.L. (at x = 4)

$$= \lim_{x \to 4^{-}} f(x) = \lim_{x \to 4^{-}} \frac{x-4}{|x-4|} + a = -1 + a$$

R H L (at
$$x = 4$$
)

$$= \lim_{x \to 4^{+}} f(x) = \lim_{x \to 4^{+}} \frac{x - 4}{|x - 4|} + b = 1 + b$$

and
$$f(4) = a + b$$

Since, f(x) is continuous at x = 4.

$$\therefore$$
 $-1 + a = a + b \implies b = -1$

and
$$1 + b = a + b$$

$$\Rightarrow a = 1$$

Thus, f(x) is continuous at x = 4 if a = 1 and b = -1.

OR

24. (b) We have, L.H.L. (at x = 5)

$$= \lim_{x \to 5^{-}} f(x) = \lim_{x \to 5^{-}} (3x - 8) = 7$$

and R.H.L. (at
$$x = 5$$
)

$$\therefore \lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (2k)$$

$$= 2k$$
 and $f(5) = 7$

Since, f(x) is continuous at x = 5.

$$\therefore \lim_{x \to 5^{-}} f(x) = \lim_{x \to 5^{+}} f(x) = f(5)$$

$$\Rightarrow$$
 7 = 2 k

$$\therefore k = \frac{7}{2}$$

25. Let
$$I = \int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)}$$

$$= \frac{1}{5} \left(\int_0^\infty \frac{1}{x^2 + 4} dx - \int_0^\infty \frac{1}{(x^2 + 9)} dx \right)$$

$$= \frac{1}{5} \left[\left[\frac{1}{2} \tan^{-1} \frac{x}{2} \right]_0^\infty - \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^\infty \right]$$

$$= \frac{1}{5} \left[\left(\frac{1}{2} \cdot \frac{\pi}{2} - 0 \right) - \left(\frac{1}{3} \cdot \frac{\pi}{2} - 0 \right) \right]$$

$$= \frac{1}{5} \left(\frac{\pi}{4} - \frac{\pi}{6} \right) = \frac{\pi}{60}$$

26. Let the monthly income of Aryan be $\stackrel{?}{\sim} 3x$ and that of Babban be $\stackrel{?}{\sim} 4x$.

Also, let monthly expenditure of Aryan be $\stackrel{?}{\sim} 5y$ and that of Babban be $\stackrel{?}{\sim} 7y$.

According to question,

$$3x - 5y = 15000$$

$$4x - 7y = 15000$$

These equations can be written as AX = B

where,
$$A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}$$
, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -5 \\ 4 & -7 \end{vmatrix} = (-21 + 20) = -1 \neq 0$$

Thus, A^{-1} exists. So, system of equations has a unique solution and given by $X = A^{-1} B$

Now,
$$adj(A) = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\operatorname{adj}(A)}{|A|} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

Now,
$$X = A^{-1}B = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix}$$

 $\Rightarrow x = 30000 \text{ and } y = 15000$

So, monthly income of Aryan = 3 × 30000 = ₹90000 Monthly income of Babban = 4 × 30000 = ₹120000

27. (a) We have,
$$f(x) = \frac{a\sin x + 2\cos x}{\sin x + \cos x}$$

$$(a\cos x - 2\sin x)(\sin x + \cos x)$$

$$\Rightarrow f'(x) = \frac{-(a\sin x + 2\cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{-2\sin^2 x + a\cos^2 x + a\sin^2 x - 2\cos^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{a - 2}{(\sin x + \cos x)^2}$$

For, f(x) to be strictly increasing for all values of x, $\Rightarrow f'(x) > 0 \Rightarrow a - 2 > 0 \Rightarrow a > 2$ $(\because (\sin x + \cos x)^2 > 0)$

OR

27. (b)
$$f(x) = \frac{\log x}{x}$$

$$\Rightarrow f'(x) = -\frac{\log x}{x^2} + \frac{1}{x^2} = \frac{1 - \log x}{x^2}$$

$$f(x)$$
 is strictly increasing if $f'(x) > 0$

$$i.e., \text{ if } \frac{1 - \log x}{x^2} > 0$$

i.e., if $1 - \log x > 0$ if $\log x < 1$ *i.e.*, if x < e

Also, f(x) is defined for x > 0

 \therefore f(x) is increasing on (0, e)

28. (a) Let
$$I = \int \frac{x^2}{(ax+b)^2} dx$$

Put
$$ax + b = t \implies dx = \frac{1}{a}dt$$

$$I = \frac{1}{a^3} \int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3} \int \left(1 + \frac{b^2}{t^2} - \frac{2b}{t}\right) dt$$

$$= \frac{1}{a^3} \left(t - \frac{b^2}{t} - 2b \log t\right) + C$$

$$= \frac{1}{a^3} \left(ax + b - \frac{b^2}{ax + b} - 2b \log(ax + b)\right) + C$$

OR

28. (b) Let
$$I = \int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Let $a^2 \sin^2 x + b^2 \cos^2 x = t \implies (a^2 - b^2) \sin 2x \, dx = dt$

$$\therefore I = \frac{1}{(a^2 - b^2)} \int_{t}^{1} dt = \frac{1}{(a^2 - b^2)} \log|t| + C$$

$$\Rightarrow I = \frac{1}{(a^2 - b^2)} \log |a^2 \sin^2 x + b^2 \cos^2 x| + C$$

29. Let the equation of line passing through (2, 1, 3) and perpendicular to the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \text{ and } \frac{x}{-3} = \frac{y}{2} = \frac{z}{5} \text{ be}$$

$$\frac{x-2}{l} = \frac{y-1}{m} = \frac{z-3}{n} \qquad \dots (i)$$

Since, line (i) is $\perp r$ to given lines.

$$\therefore l \cdot 1 + m \cdot 2 + n \cdot 3 = 0$$

and
$$l \cdot (-3) + m \cdot 2 + n \cdot 5 = 0$$

$$\Rightarrow \frac{l}{10-6} = \frac{m}{-9-5} = \frac{n}{2+6}$$

$$\Rightarrow \frac{l}{2} = \frac{m}{-7} = \frac{n}{4}.$$

 \therefore Equation of the required line is

$$\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$
.

Also its vector equation is

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k}).$$

30. (a) Given,
$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$
 and $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

Now,
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= (-2-9)\hat{i} - (4-3)\hat{j} + (6+1)\hat{k} = -11\hat{i} - \hat{j} + 7\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-11)^2 + (-1)^2 + 7^2} = \sqrt{171} = 3\sqrt{19}$$

Also,
$$|\vec{a}| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$

and
$$|\vec{b}| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$$

Let θ be the angle between \vec{a} and \vec{b} .

Then,
$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{3\sqrt{19}}{(\sqrt{14})(\sqrt{14})} = \frac{3}{14}\sqrt{19}$$

OF

30. (b) Let $\vec{\alpha}$ be the position vector of point P(1, 2, 3)

$$\vec{\alpha} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Given line in vector form is

$$\vec{r} = 6\hat{i} + 7\hat{i} + 7\hat{k} + \lambda(3\hat{i} + 2\hat{i} - 2\hat{k})$$

$$\vec{a} = 6\hat{i} + 7\hat{j} + 7\hat{k}$$
 and $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Now,
$$\vec{\alpha} - \vec{a} = -5\hat{i} - 5\hat{j} - 4\hat{k}$$
, $|\vec{\alpha} - \vec{a}|^2 = 66$

$$(\vec{\alpha} - \vec{a}) \cdot \vec{b} = -15 - 10 + 8 = -17, |\vec{b}| = \sqrt{9 + 4 + 4} = \sqrt{17}$$

$$\therefore$$
 Required distance = $\sqrt{66 - \left[\frac{-17}{\sqrt{17}}\right]^2} = 7$ units

31. Let E = the event of A speaking the truth and F = the event of B speaking the truth

Then,
$$P(E) = \frac{60}{100} = \frac{3}{5}$$
 and $P(F) = \frac{90}{100} = \frac{9}{10}$

Required probability = P (A and B contradicting each other)

$$= P(E\overline{F} \text{ or } \overline{E}F) = P(E\overline{F}) + P(\overline{E}F)$$
$$= P(E) \cdot P(\overline{F}) + P(\overline{E}) \cdot P(F)$$

[: E and F are independent events]

$$= P(E) \cdot [1 - P(F)] + [1 - P(E)] \cdot P(F)$$

$$= \frac{3}{5} \left(1 - \frac{9}{10} \right) + \left(1 - \frac{3}{5} \right) \cdot \frac{9}{10} = \frac{21}{50} = \frac{42}{100}$$

Thus, *A* and *B* are likely to contradict each other in 42% cases.

32. (a) Here,
$$f: A \to B$$
 is given by $f(x) = \frac{x-1}{x-2}$,

where
$$A = R - \{2\}$$
 and $B = R - \{1\}$

Let
$$f(x_1) = f(x_2)$$

where $x_1, x_2 \in A$ (i.e., $x_1 \neq 2, x_2 \neq 2$)

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2} \Rightarrow (x_1 - 1)(x_2 - 2) = (x_1 - 2)(x_2 - 1)$$

$$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$$

$$\Rightarrow$$
 $-2x_1 - x_2 = -x_1 - 2x_2 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let
$$y \in B = R - \{1\}$$
 i.e., $y \in R$ and $y \neq 1$

such that f(x) = y

$$\Leftrightarrow \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x-1$$

$$\Leftrightarrow xy - 2y = x - 1 \Leftrightarrow x(y - 1) = 2y - 1$$

$$\Leftrightarrow x = \frac{2y-1}{y-1}$$

$$\therefore f(x) = y \text{ when } x = \frac{2y-1}{y-1} \in A \text{ (as } y \neq 1)$$

Hence, *f* is onto.

Thus, *f* is one–one and onto.

32. (b) Given relation is $R = \{(a, b): a \le b^2\}$ Reflexivity: Let $a \in \text{real numbers}$.

 $aRa \Rightarrow a \leq a^2$

but if a < 1, then $a \not< a^2$

For example, $a = \frac{1}{2} \implies a^2 = \frac{1}{4}$ so, $\frac{1}{2} \nleq \frac{1}{4}$

Hence, *R* is not reflexive.

Symmetricity: $aRb \Rightarrow a \leq b^2$

But then $b \le a^2$ is not true \mathbb{R}

∴ aRb ≠ bRa

For example, a = 2, b = 5

then $2 \le 5^2$ but $5 \le 2^2$ is not true.

Hence, *R* is not symmetric.

Transitivity : Let $a, b, c \in \mathbb{R}$

Considering aRb and bRc

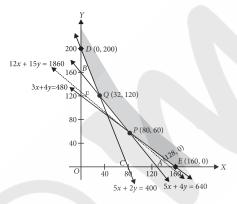
$$aRb \implies a \le b^2 \text{ and } bRc \implies b \le c^2 \implies a \le c^4 \implies aRc$$

For example, if a = 2; b = -3, c = 1

 $aRb \Rightarrow 2 \le 9$; $bRc \Rightarrow -3 \le 1$; $aRc \Rightarrow 2 \le 1$ is not true.

Hence, R is not transitive

33. (a)



The shaded region *DQPE* represents the feasible region of the given LPP.

The corner points of the feasible region are

D(0, 200), Q(32, 120), P(80, 60) and E(160, 0)

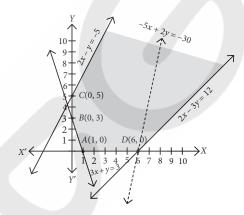
The values of the objective function at these points are:

Corner Points	Value of $Z = 12000x + 15000y$
D(0, 200)	3000000
Q(32, 120)	2184000
P(80, 60)	1860000 (Minimum)
E(160,0)	1920000

From the table, we can observe that 1860000 is the minimum value of Z at P(80, 60). Since the region is unbounded we have to check that the inequality 12000x + 15000y < 1860000 in open half plane has any point in common or not. Since it has no point in common. So, minimum value of Z is at P(80, 60) and the minimum value of Z is 1860000.

OF

33. (b) The feasible region (shaded) is shown in the figure.



We can observe that the feasible region is unbounded. We now evaluate Z at the corner points.

Corner Points	Value of $Z = -50x + 20y$
(0, 5)	100
(0, 3)	60
(1,0)	-50
(6, 0)	-300 (Minimum)

From this table, we find that -300 is the smallest value of Z at the corner point (6, 0).

Here, the feasible region is unbounded. Therefore, – 300 may or may not be the minimum value of Z. To check, we graph the inequality – 50x + 20y < -300 i.e., – 5x + 2y < -30 and check whether the resulting open half plane has points in common with feasible region or not. If it has common points, then –300 will not be the minimum value of Z. Otherwise, – 300 will be the minimum value of Z. As observe from figure, it has common points. Therefore, Z = -50x + 20y has no minimum value subject to the given constraints.

34. We have,
$$y^2 dx + (x^2 - xy + y^2) dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$$

This is a homogeneous differential equation.

$$\therefore$$
 Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$
, we get

$$v + x \frac{dv}{dx} = \frac{-v^2 x^2}{x^2 - v x^2 + v^2 x^2} = \frac{-v^2}{1 - v + v^2}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{-v^2}{1 - v + v^2} - v \Rightarrow x \frac{dv}{dx} = \frac{-v - v^3}{1 - v + v^2}$$
$$\Rightarrow \frac{1 - v + v^2}{v(1 + v^2)} dv = -\frac{1}{x} dx$$

Integrating both sides, we get

$$\int \frac{1+v^2}{v(1+v^2)} dv - \int \frac{v}{v(1+v^2)} dv = -\int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{1}{v} dv - \int \frac{1}{1+v^2} dv = -\int \frac{1}{x} dx$$

$$\Rightarrow \log |v| - \tan^{-1}v = -\log |x| + \log C$$

$$\Rightarrow \log \left| \frac{vx}{C} \right| = \tan^{-1}v \Rightarrow \left| \frac{vx}{C} \right| = e^{\tan^{-1}v}$$

$$\Rightarrow y = Ce^{\tan^{-1}(y/x)}$$
 is the required solution.

35. Let
$$I = \int_{0}^{\frac{3}{2}} |x \cos \pi x| dx$$

$$|x\cos\pi x| = \begin{cases} x\cos\pi x ; & 0 < x < \frac{1}{2} \\ -x\cos\pi x ; & \frac{1}{2} < x < \frac{3}{2} \end{cases}$$

$$\therefore I = \int_{0}^{\frac{1}{2}} (x \cos \pi x) dx - \int_{\frac{1}{2}}^{\frac{3}{2}} (x \cos \pi x) dx$$

$$= \left[\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_0^{\frac{1}{2}} - \left[\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \left[\left(\frac{1}{2\pi} + 0 \right) - \left(0 + \frac{1}{\pi^2} \right) \right] - \left[\left(\frac{-3}{2\pi} + 0 \right) - \left(\frac{1}{2\pi} + 0 \right) \right]$$

$$=\frac{1}{2\pi}-\frac{1}{\pi^2}+\frac{4}{2\pi}=\frac{5}{2\pi}-\frac{1}{\pi^2}=\frac{5\pi-2}{2\pi^2}$$

36. Let *B*, *R*, *Y* and *G* denote the events that ball drawn is blue, red, yellow and green respectively.

$$P(B) = \frac{12}{35}, P(R) = \frac{8}{35}, P(Y) = \frac{10}{35} \text{ and } P(G) = \frac{5}{35}$$

(i)
$$P(G \cap B) = P(B).P(G \mid B)$$

$$=\frac{12}{35}\cdot\frac{5}{34}=\frac{6}{119}$$

(ii)
$$P(R \cap Y) = P(Y).P(R \mid Y)$$

$$= \frac{10}{35} \cdot \frac{8}{34} = \frac{8}{119}$$

(iii)(a) Let E = event of drawing a first red ball and E = event of drawing a second red ball

Here,
$$P(E) = \frac{8}{35}$$
 and $P(F) = \frac{7}{34}$

:.
$$P(F \cap E) = P(E).P(F \mid E) = \frac{8}{35} \cdot \frac{7}{34} = \frac{4}{85}$$

OF

(iii)(b)
$$P(Y' \cap G) = P(G) \cdot (Y' \mid G) = \frac{5}{35} \cdot \frac{24}{34} = \frac{12}{119}$$

37. (i) The coordinates of A are (2, 2).

$$\therefore$$
 P.V. of $A = 2\hat{i} + 2\hat{j}$

Here, (5, 3) are the coordinates of B.

$$\therefore \text{ P.V. of } B = 5\hat{i} + 3\hat{j}$$

(ii) The coordinates of C are (6, 5).

$$\therefore$$
 P.V. of C = $6\hat{i} + 5\hat{j}$

Here, (9, 8) are the coordinates of D.

$$\therefore$$
 P.V. of $D = 9\hat{i} + 8\hat{j}$

(iii)(a) P.V. of
$$B = 5\hat{i} + 3\hat{j}$$
 and P.V. of $C = 6\hat{i} + 5\hat{j}$

$$\therefore \overrightarrow{BC} = (6-5)\hat{i} + (5-3)\hat{j} = \hat{i} + 2\hat{j}$$

OR

(iii)(b) Since P.V. of
$$A = 2\hat{i} + 2\hat{j}$$
, P.V. of $D = 9\hat{i} + 8\hat{j}$

$$\overrightarrow{AD} = (9-2)\hat{i} + (8-2)\hat{j} = 7\hat{i} + 6\hat{j}$$
$$|\overrightarrow{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85 \implies |\overrightarrow{AD}| = \sqrt{85} \text{ units}$$

38. (i) Let *A* be the 2×3 matrix representing the annual sales of products in two markets.

$$A = \begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$$
Market II

Let *B* be the column matrix representing the sale price of each unit of products *x*, *y*, *z*.

$$B = \begin{bmatrix} 2.5 \\ 1.5 \end{bmatrix}$$

Now, reverlue = sale price \times number of items sold

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

Therefore, the revenue collected from Market I = 3 46000

(ii) Let C be the column matrix representing cost price of each unit of products x, y, z.

Then,
$$C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

:. Total cost in each market is given by

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Now, Profit matrix = Revenue matrix - Cost matrix

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets

