MATHEMATICS SAMPLE)UESTION APER



General Instructions:

Read the following instructions very carefully and strictly follow them:

- This Question paper contains 38 questions. All questions are compulsory.
- This Question paper is divided into five Sections A, B, C, D and E.
- In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
- In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
- In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
- (vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
- (vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
- Use of calculator is not allowed.

Time Allowed: 3 hours

SECTION - A

This section comprises of multiple choice questions (MCQs) of 1 mark each.

Select the correct option (Question 1 - Question 18)

1. Find:
$$\int \frac{\sin^6 x}{\cos^8 x} dx$$

(a)
$$\frac{1}{7} \tan^7 x + C$$

(b)
$$\frac{1}{5} \tan^5 x + C$$

(c)
$$\frac{1}{4} \tan^4 x + C$$
 (d) $\frac{1}{6} \tan^6 x + C$

(d)
$$\frac{1}{6} \tan^6 x + 0$$

- 2. The value of k which makes $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ $\frac{dy}{dx} + y = \cos x \sin x$ is (a) e^{-x} (b) e^{x} (c) e^{2x}
 - (a) 8

- (c) -1
- (d) none of these
- **3.** The function $f: N \rightarrow N$ defined by

Maximum Marks: 80

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}, \text{ is}$$

- (a) bijective
- (b) one-one but not onto
- (c) onto but not one-one
- (d) neither one-one nor onto
- (a) $\frac{1}{7}\tan^7 x + C$ (b) $\frac{1}{5}\tan^5 x + C$ **4.** If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then (A 2I)(A 3I) is equal to
- (a) A (b) I (c) 5I
- (d) O
- 5. The integrating factor of the differential equation

- **6.** The value of $(\vec{a} \cdot \hat{i})^2 + (\vec{a} \cdot \hat{j})^2 + (\vec{a} \cdot \hat{k})^2$ is
 - (a) $|\vec{a}|^2$
- (c) \vec{a}^2
- (d) None of these

- 7. If $x = a\sec\theta$, $y = b\tan\theta$, then $\frac{dy}{dx}$ is

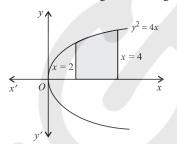
 - (a) $\frac{b}{a} \csc \theta$ (b) $\frac{-b}{a} \csc \theta$

 - (c) $\frac{a}{b} \csc \theta$ (d) $\frac{-a}{b} \csc \theta$
- 8. The range of $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$ is
 - (a) $[0, \pi]$
- (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
- (c) $[0, \pi)$
- (d) $\left| 0, \frac{\pi}{2} \right|$
- 9. Find the degree and order of the differential equation: $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$.
 - (a) 2, 1
- (c) 0, 1
- (d) 3, 2
- **10.** Let *R* be a relation on the set *N* defined by $\{(x, y) : x, y \in \mathbb{N}, 2x + y = 41\}$. Then, R is
 - (a) Reflexive
- (b) Symmetric
- (c) Transitive
- (d) None of these
- 11. Evaluate : $\int_{1}^{2} \frac{dx}{x(1 + \log x)^2}$
 - (a) $\log 3/(1 + \log 3)$
- (b) $\log 2/(1 + \log 2)$
- (c) $\log 3/(1 + \log 2)$
- (d) $\log 5/(1 + \log 5)$
- 12. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.
 - (a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$
- **13.** Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix C = 5A+3B is
 - (a) 3×5 and m = n
- (b) 3×5 and $m \neq n$
- (c) 3×3
- (d) 5×5
- **14.** If A(adj A) = 5I, where I is identity matrix of order 3, then |adj A| =
 - (a) 125
- (b) 25
- (c) 10
- **15.** The area of the ellipse $\frac{x^2}{12^2} + \frac{y^2}{12^2} = 1$ is

- (a) 156π sq. units
- (b) 150π sq. units
- (c) 155π sq. units
- (d) 170π sq. units
- **16.** The value of x for which $(x x^2)$ is maximum, is
 - (a) 3/4
- (b) 1/2
- (c) 1/3
- 17. If $f(x) = -\sqrt{25 x^2}$, then $\lim_{x \to 1} \frac{f(x) f(1)}{x 1}$ is equal to

 (a) $\frac{1}{24}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) $\frac{1}{\sqrt{24}}$

- 18. Find the area of shaded region of the given graph.



- (a) $\frac{4}{3}(8+2\sqrt{2})$ sq. units (b) $\frac{4}{3}(8-2\sqrt{2})$ sq. units
- (c) $\frac{3}{4}(8-2\sqrt{2})$ sq. units (d) $\frac{3}{4}(8+2\sqrt{2})$ sq. units

ASSERTION-REASON BASED QUESTIONS

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
- (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
- (c) (A) is true but (R) is false.
- (d) (A) is false but (R) is true.
- 19. Assertion (A): The vector perpendicular to both

lines
$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$$
 and $L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ is $-\hat{i} - 7\hat{j} + 5\hat{k}$.

Reason (R): The vector perpendicular to the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is $\vec{b}_1 \times \vec{b}_2$.

20. Assertion (A): If $f'(x) = (x-1)^3 (x-2)^8$, then f(x)has neither maximum nor minimum at x = 2.

Reason (R): If f'(x) changes sign from negative to positive as x increases through a, then a is a point of local maxima.

SECTION - B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. If
$$x = \frac{1-t^2}{1+t^2}$$
 and $y = \frac{2t}{1+t^2}$, then find $\frac{dy}{dx}$.

22. (a) If a line makes angles 90° , 60° and 30° with the positive directions of x, y and z-axis respectively, then find its direction cosines.

OR

- 22. (b) Find the direction cosines of the line passing through two points (2, 1, 0) and (1, -2, 3).
- 23. Show that $y = cx + \frac{a}{c}$ is a solution of differential equation $y = x \frac{dy}{dx} + \frac{c}{a} \frac{dy}{dx}$.
- **24.** (a) Find the values of *a* and *b* such that the function

$$f \text{ defined by } f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a+b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$$

OR

24. (b) Consider
$$f(x) = \begin{cases} 3x - 8, & \text{if } x \le 5 \\ 2k, & \text{if } x > 5 \end{cases}$$

Find the value of k, if f(x) is continuous at x = 5.

25. Evaluate :
$$\int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)}$$

SECTION - C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

- 26. The monthly incomes of Aryan and Babban are in the ratio 3: 4 and their monthly expenditures are in the ratio 5: 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method.
- 27. (a) For what values of x, function $f(x) = \frac{a \sin x + 2 \cos x}{\sin x + \cos x}$ is strictly increasing for all values of x?

OR

- 27. (b) Find the interval in which $f(x) = \frac{\log x}{x}$ is strictly increasing.
- **28.** (a) Evaluate: $\int x^2 (ax+b)^{-2} dx$

OR

28. (b) Evaluate:
$$\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

- 29. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.
- **30.** (a) Find the sine of the angle between the vectors $\vec{a} = 2\hat{i} \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$.

OF

- **30.** (b) Find the distance of the point (1, 2, 3) from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.
- **31.** *A* speaks truth in 60% of the cases and *B* in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

SECTION - D

This section comprises of 4 long answer (LA) type questions of 5 marks each.

32. (a) Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto.

OR

- **32.** (b) Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive
- 33. (a) Minimise Z = 12000x + 15000ySubject to constraints $5x + 4y \ge 640$, $5x + 2y \ge 400$, $3x + 4y \ge 480$; $x, y \ge 0$.

33. (b) Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints:

$$2x - y \ge -5$$

$$3x + y \ge 3$$

$$2x - 3y \le 12$$

$$x \ge 0, y \ge 0$$

34. Solve the following differential equation :

$$y^2 dx + (x^2 - xy + y^2) dy = 0$$

35. Evaluate: $\int_{0}^{\frac{3}{2}} |x \cos \pi x| dx$

SECTION - E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each.

Case Study 1

36. In a play zone, Aastha is playing crane game. It has 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. Aastha draws two balls one after the other without replacement.



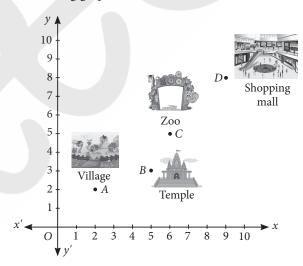
- On the basis of above information, answer the following questions.
- (i) What is the probability that the first ball is blue and the second ball is green?
- (ii) What is the probability that the first ball is yellow and the second ball is red?
- (iii)(a) What is the probability that both the balls are red?

OR

(iii)(b) What is the probability that the first ball is green and the second ball is not yellow?

Case Study 2

37. Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.



- On the basis of above information, answer the following questions.
- (i) Find the Position vector of *A* and *B*.
- (ii) Find the Position vector of *C* and *D*.
- (iii)(a) Find the vector \overrightarrow{BC} in terms of \hat{i} , \hat{j} .

OR

(iii)(b) Find the Length of vector \overrightarrow{AD} .

Case Study 3

38. A manufacturer produces three types of bolts, x, y and z which he sells in two markets. Annual sales (in \mathfrak{T}) are indicated below:



Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

If unit sales prices of x, y and z are $\ref{2.50}$, $\ref{1.50}$ and $\ref{1.00}$ respectively.

On the basis of above information, answer the following questions.

- (i) Find the total revenue collected from the Market-I.
- (ii) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively, then find the gross profit from both the markets.