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MATHEMATICS

STANDARD

Sample Question Paper

SQP 9

BLUEPRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Unit/Chapter		MCQs (1 mark)	VSA (2 marks)	SA (3 marks)	LA (5 marks)	Case Based (4 marks)	Total
I.	**Number Systems**							
	1.	Real Numbers	1(1)	–	–	1(5)	–	**2(6)**
II.	**Algebra**							
	2.	Polynomials	1(1)	–	–	–	–	**8(20)**
	3.	Pair of Linear Equations in Two Variables	1(1)	–	1(3)	1(5)	–	
	4.	Quadratic Equations	–	1(2)	1(3)*	–	–	
	5.	Arithmetic Progressions	1(1)	–	–	–	1(4)*	
III.	**Coordinate Geometry**							
	6.	Coordinate Geometry	3(3)	–	1(3)*	–	–	**4(6)**
IV.	**Geometry**							
	7.	Triangles	2(2)	1(2)*	–	–	1(4)*	**8(15)**
	8.	Circles	2(2)	1(2)*	1(3)	–	–	
V.	**Trigonometry**							
	9.	Introduction to Trigonometry	2(2)	1(2)	–	–	–	**5(12)**
	10.	Some Applications of Trigonometry	–	–	1(3)	1(5)*	–	
VI.	**Mensuration**							
	11.	Areas Related to Circles	1(1)	–	–	–	1(4)*	**5(10)**
	12.	Surface Areas and Volumes	2(2)	–	1(3)	–	–	
VII.	**Statistics and Probability**							
	13.	Statistics	1(1)	–	–	1(5)*	–	**6(11)**
	14.	Probability	3(3)	1(2)	–	–	–	
	Total		**20(20)**	**5(10)**	**6(18)**	**4(20)**	**3(12)**	**38(80)**

*It is a choice based question.

Subject Code : **041**

SQP-9

Mathematics - Standard

Time Allowed : 3 Hours ***Maximum Marks : 80***

General Instructions :

Read the following instructions carefully and follow them :

1. *This question paper contains 38 questions.*
2. *This Question Paper is divided into 5 Sections A, B, C, D and E.*
3. *In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion- Reason based questions of 1 mark each.*
4. *In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.*
5. *In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.*
6. *In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.*
7. *In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.*
8. *All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.*
9. *Draw neat and clean figures wherever required.*
10. *Take $\pi = 22/7$ wherever required if not stated.*
11. *Use of calculators is not allowed.*

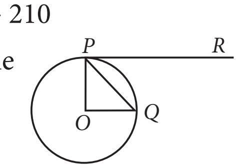
SECTION A

Section A consists of 20 questions of 1 mark each.

1. The sum of first 10 terms of the A.P. $x - 8, x - 2, x + 4, \ldots$ is

(a) $10x + 210$ (b) $10x + 190$ (c) $5x + 190$ (d) $5x + 210$

2. If O is centre of a circle and chord PQ makes an angle 50° with the tangent PR at the point of contact P, find the angle made by the chord at the centre.



(a) 130° (b) 100° (c) 50° (d) 30°

3. If $\Delta ABC \sim \Delta PQR$, perimeter of ΔABC = 20 cm, perimeter of ΔPQR = 40 cm and PR = 8 cm, then the length of AC is

(a) 8 cm (b) 6 cm (c) 4 cm (d) 5 cm

4. A letter is chosen at random from the English alphabet. Probability that it is a letter of the word "SIMULTANEOUSLY" is

(a) $\frac{14}{26}$ (b) $\frac{11}{26}$ (c) $\frac{10}{26}$ (d) $\frac{15}{26}$

5. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part is half the conical part, then find the ratio of the radius and the height of the conical part is

(a) $\frac{1}{\sqrt{15}}$ (b) $\frac{1}{\sqrt{17}}$ (c) $\frac{1}{\sqrt{13}}$ (d) $\frac{1}{\sqrt{19}}$

6. For some data $x_1, x_2, \ldots, x_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the value of $\sum_{1}^{n} f_i\left(x_i - \overline{x}\right)$ is equal to

(a) $n\overline{x}$ (b) 1 (c) Σf_i (d) 0

7. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 4 and –3, then $a - b$ is
(a) 12 (b) 10 (c) 7 (d) 1

8. The value of k, if the distance between $A(k, 3)$ and $B(2, 3)$ is 5 units, is
(a) 5 (b) 6 (c) 7 (d) 8

9. If $\cos(\alpha + \beta) = 0$, then value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is equal to
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 0 (d) $\sqrt{2}$

10. A card is drawn from a well - shuffled pack of cards. The probability that it will be a black queen is
(a) $\frac{1}{13}$ (b) $\frac{1}{26}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

11. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have
(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution

12. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (–2, 5), then the coordinates of the other end of the diameter are
(a) (–6, 7) (b) (6, –7) (c) (6, 7) (d) (–6, –7)

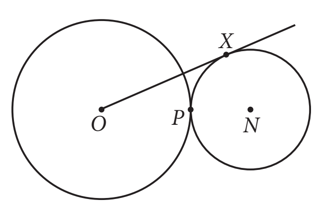
13. If $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$, then $\tan(\theta + \phi)$ is
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) not defined

14. In two triangles ABC and DEF, $\angle A = \angle E$ and $\angle B = \angle F$. Then, $\frac{AB}{AC}$ is equal to
(a) $\frac{DE}{DF}$ (b) $\frac{ED}{EF}$ (c) $\frac{EF}{ED}$ (d) $\frac{EF}{DF}$

15. If an arc subtending an angle of 60° at the centre of a circle A and another arc subtending an angle of 90° at the centre of circle B, are of same length, then the ratio of area of circle A to that of circle B is
(a) 11 : 15 (b) 11 : 25 (c) 9 : 4 (d) 36 : 16

16. The smallest number which when increased by 17 is exactly divisible by 520 and 468 is
(a) 4680 (b) 4663 (c) 4860 (d) 4636

17. Two circles with centres O and N touch each other at point P as shown. O, P and N are collinear. The radius of the circle with centre O is twice that of the circle with centre N. OX is a tangent to the circle with centre N, and $OX = 18$ cm.



(Note : The figure is not to scale.)

What is the radius of the circle with centre N?
(a) $\frac{18}{\sqrt{2}}$ cm (b) 9 cm (c) $\frac{9}{\sqrt{2}}$ cm (d) $\frac{18}{\sqrt{10}}$ cm

18. What is the total surface area of a solid hemisphere of diameter 'd'?
(a) $3\pi d^2$ (b) $2\pi d^2$ (c) $\frac{1}{2}\pi d^2$ (d) $\frac{3}{4}\pi d^2$

DIRECTION : In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Assertion (A) :** Point $P\left(1, \frac{5}{2}\right)$ is equidistant from the points $A(-5, 3)$ and $B(7, 2)$.

Reason (R) : If a point P is equidistant from the points A and B, then $AP = BP$.

20. **Assertion (A) :** Two players, Sania and Ashnam play a tennis match. The probability of Sania winning the match is 0.79 and that of Ashnam winning the match is 0.21.

Reason (R) : The sum of probabilities of two complementary events is 1.

SECTION B

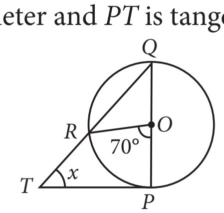
Section B consists of 5 questions of 2 marks each.

21. If 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$ and the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots, find the value of b.

22. If $A = 60°$ and $B = 30°$, verify that :
$\sin (A + B) = \sin A \cos B + \cos A \sin B$

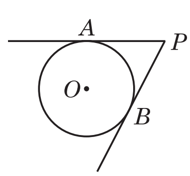
23. 14 cards numbered 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 are placed in a box and mixed thoroughly. If a card is drawn from the box, then find the probability that the number on the card divisible by 3 or 2.

24. (a) In the adjoining figure, if PQ is diameter and PT is tangent, then find x.

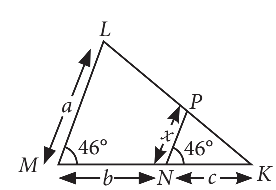


OR

(b) In the given figure, $PB = 24$ cm, $OP = 25$ cm, PA and PB are tangents of the circle. Find the length of PA and OB.



25. (a) In figure, $\angle LMK = \angle PNK = 46°$. Express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



OR

(b) In ΔABC, $\angle CAB = 90°$ and $AD \perp BC$. If $AC = 75$ cm, $AB = 1$ m and $BD = 80$ cm, find AD.

SECTION C

Section C consists of 6 questions of 3 marks each.

26. (a) Solve for x : $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$; $x \neq 0, 1, 2$

OR

(b) ₹ 6500 is divided equally among a certain number of persons. If there are 15 more persons, each will get ₹ 30 less. Find the original number of persons.

27. For which value(s) of λ, the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ has
(i) no solution ?
(ii) infinitely many solutions ?

28. (a) Show that $A(6, 4)$, $B(5, -2)$ and $C(7, -2)$ are the vertices of an isosceles triangle. Also, find the length of the median through A.

OR

(b) If $C(-2, 3)$ is equidistant from $A(3, -1)$ and $B(x, 8)$, then find x. Also, find the distances BC and AB.

29. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 15 cm, and its base is of radius 2.8 cm, then find the volume of wood in the toy.

30. Two vertical poles of different heights are standing 20 m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. (Take $\sqrt{3} = 1.73$)

31. In two concentric circles, a chord of the larger circle touches the smaller circle. If the length of this chord is 8 cm and the diameter of the smaller circle is 6 cm, then find the diameter of the larger circle.

SECTION D

Section D consists of 4 questions of 5 marks each.

32. Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

33. By comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

(i) $3x + y - 14 = 0$
$2x + 5y - 5 = 0$

(ii) $5x + 3y - 15 = 0$
$10x + 6y - 30 = 0$

(iii) $5x - y + 7 = 0$
$15x - 3y + 17 = 0$

34. (a) The following table gives weekly wages in rupees of workers in a certain commercial organization. The frequency of class 49-52 is missing. It is known that the mean of the frequency distribution is 47.2. Find the missing frequency.

Weekly wages (in ₹)	40-43	43-46	46-49	49-52	52-55
Number of workers	31	58	60	?	27

OR

(b) Find the mode of the following data.

Class - interval	0-9	10-19	20-29	30-39	40-49	50-59
Frequency	12	15	21	17	19	6

35. (a) A pole 6 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point P on the ground is 60° and the angle of depression of the point P from the top of the tower is 45°. Find the height of the tower and the distance of point P from the foot of the tower. (Use $\sqrt{3} = 1.73$)

OR

(b) From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

SECTION E

Section E consists of 3 case study based questions of 4 marks each.

36. Upasana bought a wall clock to gift her friend Pratibha on her birthday. The clock contains a small pendulum of length 15 cm. The minute hand and hour hand of the clock are 10 cm and 7 cm long respectively.

On the basis of the above information, answer the following questions.

(i) If the pendulum covers distance of 44 cm in one complete round, then find the angle described by pendulum at the centre.

(ii) Find the angle described by hour hand in 30 minutes.

(iii) (a) Find the area swept by the minute hand in 35 minutes.

OR

(iii) (b) Find the area swept by the hour hand in 1 hour and the area swept by the hour hand between 9 a.m. and 5 p.m.

37. Amit starts a new bakery shop. To display the cakes, he wants to put 3 cakes in 1st row, 5 cakes in 2nd row, 7 cakes in 3rd row and so on.

On the basis of the above information, answer the following questions.

(i) Find the difference of number of cakes in 17th row and 10th row.

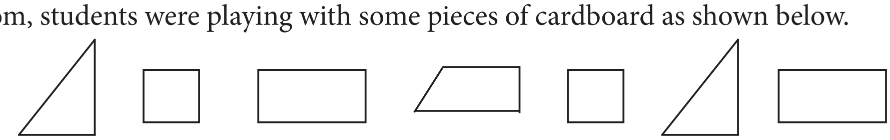
(ii) If he puts a total of 120 cakes, then how many rows are required?

(iii) (a) Find the total number of cakes in 5th and 8th row. Also, find the cakes in 30th row.

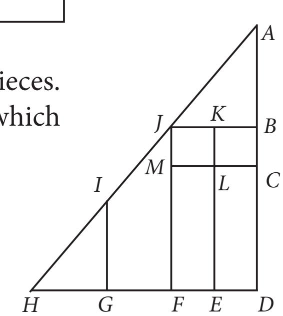
OR

(iii) (b) On next day, he arranges x cakes in 15 rows, then find the value of x.

38. In a classroom, students were playing with some pieces of cardboard as shown below.



All of a sudden, teacher entered into classroom. She told students to arrange all pieces. On seeing this beautiful image, she observed that ΔADH is right angled triangle, which contains



(i) right triangles ABJ and IGH. (ii) quadrilateral $GFJI$

(iii) squares $JKLM$ and $LCBK$ (iv) rectangles $MLEF$ and $LCDE$.

On the basis of the above information, answer the following questions.

(i) If $\angle ABJ = 90°$ and $\Delta ABJ \sim \Delta ADH$, then which similarity criterion is used?

(ii) If $\angle JFH = 90°$, $AD = 12$ m, $MF = 4$ m, $AH = 8$ m and $JH = 4$ m, then find the length of JM.

(iii) (a) If $IG \,||\, AD$, $AI = 2y$, $IH = y + 3$, $HG = y$ and $DG = 2y - 1$, then find the value of y.

OR

(iii) (b) If $\angle ABJ = 90°$, $AD = 5$ cm, and $AH = 7$ m, then find $\dfrac{AB}{AJ}$.

Self Evaluation Sheet

*Once you complete **SQP-9**, check your answers with the given solutions and fill your marks in the marks obtained column according to the marking scheme. Performance Analysis Table given at the bottom will help you to check your readiness.*

Q.No.	Chapter	Marks Per Question	Marks Obtained
1	Arithmetic Progressions	1	
2	Circles	1	
3	Triangles	1	
4	Probability	1	
5	Surface Areas and Volumes	1	
6	Statistics	1	
7	Polynomials	1	
8	Coordinate Geometry	1	
9	Introduction to Trigonometry	1	
10	Probability	1	
11	Pair of Linear Equations in Two Variables	1	
12	Coordinate Geometry	1	
13	Introduction to Trigonometry	1	
14	Triangles	1	
15	Areas Related to Circles	1	
16	Real Numbers	1	
17	Circles	1	
18	Surface Areas and Volumes	1	
19	Coordinate Geometry	1	
20	Probability	1	
21	Quadratic Equations	2	
22	Introduction to Trigonometry	2	
23	Probability	2	
24	Circles / Circles	2	
25	Triangles / Triangles	2	
26	Quadratic Equations / Quadratic Equations	3	
27	Pair of Linear Equations in Two Variables	3	
28	Coordinate Geometry / Coordinate Geometry	3	
29	Surface Areas and Volumes	3	
30	Some Applications of Trigonometry	3	
31	Circles	3	
32	Real Numbers	5	
33	Pair of Linear Equations in Two Variables	5	
34	Statistics / Statistics	5	
35	Some Applications of Trigonometry / Some Applications of Trigonometry	5	
36	Areas Related to Circles	1 + 1 + 2	
37	Arithmetic Progressions	1 + 1 + 2	
38	Triangles	1 + 1 + 2	
	Total	**80**	....
		Percentage	.............%

Performance Analysis Table

If your marks is

> 90%	**TREMENDOUS!**	➢ *You are done! Keep on revising to maintain the position.*
81-90%	**EXCELLENT!**	➢ *You have to take only one more step to reach the top of the ladder. Practise more.*
71-80%	**VERY GOOD!**	➢ *A little bit of more effort is required to reach the 'Excellent' bench mark.*
61-70%	**GOOD!**	➢ *Revise thoroughly and strengthen your concepts.*
51-60%	**FAIR PERFORMANCE!**	➢ *Need to work hard to get through this stage.*
40-50%	**AVERAGE!**	➢ *Try hard to boost your average score.*

SQP 9 SOLUTIONS

1. **(b) :** Here, $a = x - 8$, $d = x - 2 - x + 8 = 6$

$\therefore \quad S_{10} = \frac{10}{2}[2(x-8) + (10-1)6] = 5[2x - 16 + 54]$

$= 5(2x + 38) = 10x + 190$

2. **(b) :** Given, $\angle QPR = 50°$

$\because \quad OP = OQ$ (Radii of circle)

$\therefore \quad \angle OPQ = \angle OQP$ (Angle opposite to equal sides are equal)

Now, $\angle OPQ + \angle QPR = 90°$

[$\because$ Tangent is perpendicular to the radius through the point of contact]

$\Rightarrow \quad \angle OPQ + 50° = 90° \Rightarrow \angle OPQ = 40°$ and $\angle OQP = 40°$

Now, by angle sum property of triangle

$\angle OPQ + \angle OQP + \angle POQ = 180°$

$\Rightarrow \quad 40° + 40° + \angle POQ = 180°$

$\Rightarrow \quad \angle POQ = 180° - 80°$

$\Rightarrow \quad \angle POQ = 100°$

3. **(c) :** Since, $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \quad \frac{AC}{PR} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

$$\Rightarrow \quad \frac{AC}{8} = \frac{20}{40} \Rightarrow AC = \frac{20 \times 8}{40} = 4 \text{ cm}$$

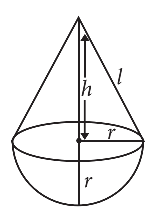
4. **(b) :** Total number of possible outcomes = 26

Distinct letters in the word "SIMULTANEOUSLY" are {S, I, M, U, L, T, A, N, E, O, Y} *i.e.*, 11

So, favourable number of outcomes = 11

$\therefore \quad$ Required probability $= \frac{11}{26}$

5. **(a) :** Let r be the radius of cone and hemispherical part and h be the height of cone.



According to question, $2\pi r^2 = \frac{1}{2}\pi r l$,

where l is slant height of cone

$\Rightarrow \quad 4r = l \quad \Rightarrow \quad 16r^2 = l^2$

$\Rightarrow \quad 16r^2 = r^2 + h^2 \quad \Rightarrow \quad 15r^2 = h^2$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{1}{15} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{15}}$$

6. **(d) :** We know that, $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

$$\Rightarrow \quad \sum_{i=1}^{n} f_i \cdot \bar{x} = \sum_{i=1}^{n} f_i \cdot x_i \Rightarrow \quad \sum_{i=1}^{n} f_i \cdot (x_i - \bar{x}) = 0$$

7. **(b) :** It is given that, 4 and –3 are the zeroes of the polynomial $x^2 + (a+1)x + b$.

$$\therefore \quad \text{Sum of zeroes} = -\frac{(a+1)}{1} = -a - 1$$

$\Rightarrow \quad 4 + (-3) = -a - 1 \Rightarrow 1 = -a - 1 \Rightarrow a = -2$

And product of zeroes $= b \Rightarrow (4)(-3) = b \Rightarrow b = -12$

$\therefore \quad a - b = -2 - (-12) = -2 + 12 = 10.$

8. **(c) :** Given, $AB = \sqrt{(k-2)^2 + (3-3)^2} = 5$

$\Rightarrow \quad \sqrt{(k-2)^2} = 5$

Squaring both sides, we have $(k-2)^2 = 25$

$\Rightarrow \quad k - 2 = \pm 5 \Rightarrow k = 2 \pm 5 \Rightarrow k = 7$ or -3

9. **(a) :** We have, $\cos(\alpha + \beta) = 0$

$\Rightarrow \quad \cos(\alpha + \beta) = \cos 90° \Rightarrow \alpha + \beta = 90°$

$$\therefore \quad \cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{90°}{2}\right) = \cos 45° = \frac{1}{\sqrt{2}}$$

10. **(b) :** Total number of possible outcomes = 52

$\because \quad$ There are two black queens.

So, favourable number of outcomes = 2

$$P(\text{selecting a black queen}) = \frac{2}{52} = \frac{1}{26}$$

11. **(d) :** We have, $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$

$$\text{Now, } \frac{a_1}{a_2} = -\frac{1}{3}, \frac{b_1}{b_2} = \frac{2}{-6} = -\frac{1}{3}, \frac{c_1}{c_2} = \frac{5}{1} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \quad$ No solution exists.

12. **(a) :** Let $A(x, y)$ and $B(2, 3)$ be the end points of the diameter and $C(-2, 5)$ be the centre of the circle.

Since, centre of the circle is mid point of diameter.

$$\therefore \quad \frac{2+x}{2} = -2 \Rightarrow x = -6 \text{ and } \frac{3+y}{2} = 5 \Rightarrow y = 7$$

Hence, the coordinates of the other end of the diameter are $(-6, 7)$.

13. **(a) :** $\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$ and $\sin\phi = \frac{1}{2} \Rightarrow \phi = 30°$

Now, $\tan(\theta + \phi) = \tan 60° = \sqrt{3}$

14. **(c) :** In ΔABC and ΔEFD,

$\angle A = \angle E$ and $\angle B = \angle F$ (Given)

$\Rightarrow \quad \Delta ABC \sim \Delta EFD$ (By AA similarity criterion)

$$\Rightarrow \quad \frac{AB}{EF} = \frac{AC}{ED} \Rightarrow \frac{AB}{AC} = \frac{EF}{ED}$$

15. **(c) :** Let r_1 and r_2 be the radii of circles A and B respectively.

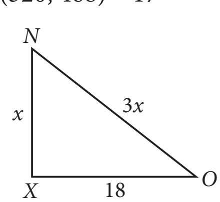
$$\therefore \quad \frac{2\pi r_1 \times 60°}{360°} = \frac{2\pi r_2 \times 90°}{360°} \Rightarrow \frac{r_1}{r_2} = \frac{90°}{60°} = \frac{3}{2}$$

$$\therefore \quad \text{Ratio of areas} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

16. **(b) :** Required number = LCM of (520, 468) – 17

= 4680 – 17 = 4663.

17. **(c) :** $(3x)^2 = x^2 + 18^2$



$9x^2 - x^2 = 324$

$$x^2 = 40 + \frac{1}{2} = \frac{81}{2} \Rightarrow x = \frac{9}{\sqrt{2}} \text{ cm}$$

18. **(d) :** Total surface area of a hemisphere = $3\pi r^2$

$= 3\pi\left(\frac{d}{2}\right)^2$ [$\because 2r = d$]

$= \frac{3}{4}\pi d^2$

19. **(a) :** We have, $P\left(1, \frac{5}{2}\right)$, $A(-5, 3)$ and $B(7, 2)$

Using distance formula, we have

$$AP = \sqrt{(1+5)^2 + \left(\frac{5}{2}-3\right)^2} = \sqrt{36+\frac{1}{4}} = \sqrt{\frac{145}{4}} = \frac{\sqrt{145}}{2} \text{ units}$$

$$BP = \sqrt{(1-7)^2 + \left(\frac{5}{2}-2\right)^2} = \sqrt{36+\frac{1}{4}} = \sqrt{\frac{145}{4}} = \frac{\sqrt{145}}{2} \text{ units}$$

Hence, $AP = BP$.

$\therefore$ Point P is equidistant from points A and B.

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

20. **(a) :** Let E_1 be the event that Sania wins and E_2 be the event that Ashnam wins.

Then E_1 and E_2 are complementary events

$\therefore$ $P(E_1) + P(E_2) = 1$

If $P(E_1) = 0.79$, then $P(E_2) = 1 - P(E_1) = 1 - 0.79 = 0.21$

$\therefore$ Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21. Since, 1 is a root of $3x^2 + ax - 2 = 0$

$\therefore$ $3(1)^2 + a(1) - 2 = 0 \Rightarrow a + 1 = 0 \Rightarrow a = -1$

Putting $a = -1$ in $a(x^2 + 6x) - b = 0$, we get

$x^2 + 6x + b = 0$ ...(i)

Since, (i) has equal roots.

$\therefore$ $D = 0$

$\Rightarrow$ $6^2 - 4(b) = 0 \Rightarrow 36 = 4b \Rightarrow b = 9$

22. We have, $A = 60°$ and $B = 30°$

$\therefore$ L.H.S. = $\sin(A + B) = \sin(60° + 30°) = \sin 90° = 1$

R.H.S. = $\sin A \cos B + \cos A \sin B$

$= \sin 60° \cos 30° + \sin 30° \cos 60°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$\Rightarrow$ L.H.S. = R.H.S. Hence proved.

23. Given cards are numbered 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 and 18.

Numbers divisible by 2 are 6, 8, 10, 12, 14, 16, 18

Numbers divisible by 3 are 6, 9, 12, 15, 18

Numbers divisible by both 2 and 3 are 6, 12, 18

$\therefore$ P(number divisible by 3 or 2) $= \frac{7+5-3}{14} = \frac{9}{14}$

24. (a) $\because$ PQ is a diameter [Given]

$\therefore$ $\angle QOR + \angle ROP = 180°$ [Linear pair]

$\Rightarrow$ $\angle QOR = 180° - 70° = 110°$

Also, $OQ = OR$ [Radii of same circle]

$\Rightarrow$ $\angle RQO = \angle ORQ$ [$\because$ Angles opposite to equal sides of triangle are equal.]

$= \frac{180° - 110°}{2} = \frac{70°}{2} = 35°$ ...(i)

Also, $QP \perp PT$ [$\because$ Tangent is perpendicular to the radius through the point of contact]

$\Rightarrow$ $\angle QPT = 90°$ ...(ii)

In ΔQPT, $\angle RQO + \angle QPT + x = 180°$

$\therefore$ $x = 180° - 90° - 35° = 55°$ [Using (i) and (ii)]

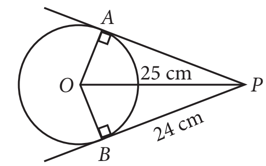
OR

(b) Since, tangents drawn from an external point are equal.

$\therefore$ $PA = PB = 24$ cm.

Also, $\angle OBP = 90°$

[Since, tangent is perpendicular to the radius through the point of contact.]



In ΔPOB, we have

$OP^2 = OB^2 + BP^2$ [By Pythagoras theorem]

$\Rightarrow$ $25^2 = OB^2 + 24^2$

$\Rightarrow$ $OB^2 = 625 - 576 = 49 \Rightarrow OB = 7$ cm

25. (a) In ΔLMK and ΔPNK,

$\angle LMK = \angle PNK$ [Each equals 46°]

$\angle K = \angle K$ [Common]

$\therefore$ $\Delta LMK \sim \Delta PNK$ [By AA similarity criterion]

$\Rightarrow$ $\frac{LM}{PN} = \frac{MK}{NK} = \frac{LK}{PK} \Rightarrow \frac{a}{x} = \frac{b+c}{c} \Rightarrow x = \frac{ac}{b+c}$

OR

(b) In ΔABD

$AB^2 = AD^2 + BD^2$

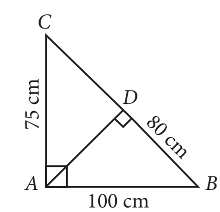
$\Rightarrow$ $(100)^2 = AD^2 + 80^2$

$\Rightarrow$ $AD^2 = 100^2 - 80^2$

$= (100 - 80)(100 + 80)$

$= 20 \times 180 = 3600$

$\therefore$ $AD = 60$ cm



26. (a) Given, $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2} \Rightarrow \frac{6x-6-2x}{x(x-1)} = \frac{1}{x-2}$

$\Rightarrow$ $\frac{4x-6}{x^2-x} = \frac{1}{x-2} \Rightarrow 4x^2 - 6x - 8x + 12 = x^2 - x$

$\Rightarrow$ $4x^2 - 14x + 12 = x^2 - x$

$\Rightarrow$ $3x^2 - 13x + 12 = 0 \Rightarrow 3x^2 - 9x - 4x + 12 = 0$

$\Rightarrow$ $3x(x - 3) - 4(x - 3) = 0 \Rightarrow (x - 3)(3x - 4) = 0$

$\Rightarrow$ $x - 3 = 0$ or $3x - 4 = 0 \Rightarrow x = 3$ or $x = 4/3$

OR

(b) Let the number of persons in 1st condition be x and in 2nd condition be $(x + 15)$.

Amount to be divided = ₹ 6500

According to the question, $\frac{6500}{x} - \frac{6500}{x+15} = 30$

$\Rightarrow$ $\frac{6500x + 97500 - 6500x}{x(x+15)} = \frac{30}{1}$

$\Rightarrow$ $30x^2 + 450x = 97500 \Rightarrow 30x^2 + 450x - 97500 = 0$

$\Rightarrow$ $x^2 + 15x - 3250 = 0 \Rightarrow x^2 + 65x - 50x - 3250 = 0$

$\Rightarrow$ $x(x + 65) - 50(x + 65) = 0 \Rightarrow (x + 65)(x - 50) = 0$

$\Rightarrow$ $x + 65 = 0$ or $x - 50 = 0 \Rightarrow x = -65$ or $x = 50$

$\because$ Number of persons can not be negative.

$\therefore$ Original number of persons = 50.

27. We have, $\lambda x + y - \lambda^2 = 0$ and $x + \lambda y - 1 = 0$
Here, $a_1 = \lambda, b_1 = 1, c_1 = -\lambda^2$ and $a_2 = 1, b_2 = \lambda, c_2 = -1$

(i) For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{-\lambda^2}{-1}$

$\Rightarrow$ $\lambda^2 - 1 = 0$ and $\lambda(\lambda - 1) \neq 0$

$\Rightarrow$ $\lambda = 1, -1$ and $\lambda \neq 0, 1$

$\therefore$ For $\lambda = -1$, system of equations has no solution.

(ii) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1} \Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} \text{ and } \frac{\lambda}{1} = \frac{\lambda^2}{1}$$

$\Rightarrow$ $\lambda^2 - 1 = 0$ and $\lambda(\lambda - 1) = 0$

$\Rightarrow$ $\lambda = 1, -1$ and $\lambda = 1, 0$

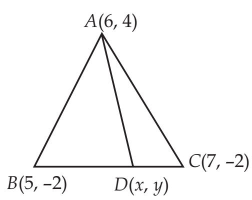
Thus, for $\lambda = 1$, system of equations has infinitely many solutions.

28. (a) Using distance formula, we have

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$AC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(5-7)^2 + (-2+2)^2} = \sqrt{4+0} = 2 \text{ units}$$



Now, $AB = AC \neq BC$.
So, ΔABC is an isosceles triangle.
Let $D(x, y)$ be the mid-point of BC.
Using mid-point formula, we have

$$(x, y) = \left(\frac{5+7}{2}, \frac{-2-2}{2}\right) = \left(\frac{12}{2}, \frac{-4}{2}\right) = (6, -2)$$

$\therefore$ Coordinates of D are $(6, -2)$

$\therefore$ Length of median, $AD = \sqrt{(6-6)^2 + (-2-4)^2}$

$= \sqrt{0+36} = 6$ units

OR

(b) Since C is equidistant from A and B.

$\therefore$ $AC = CB \Rightarrow AC^2 = CB^2$

$\Rightarrow$ $(3+2)^2 + (-1-3)^2 = (x+2)^2 + (8-3)^2$

$\Rightarrow$ $25 + 16 = x^2 + 4 + 4x + 25$

$\Rightarrow$ $x^2 + 4x - 12 = 0 \Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow$ $x(x+6) - 2(x+6) = 0 \Rightarrow (x+6)(x-2) = 0$

$\Rightarrow$ $x = -6$ or $x = 2$

Now, using distance formula

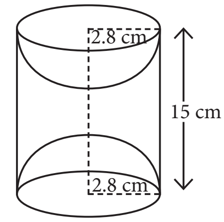
$$BC = \sqrt{(-6+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

or $\sqrt{(2+2)^2 + (8-3)^2} = \sqrt{16+25} = \sqrt{41}$ units

$$AB = \sqrt{(-6-3)^2 + (8+1)^2} = \sqrt{81+81} = 9\sqrt{2} \text{ units}$$

or $\sqrt{(2-3)^2 + (8+1)^2} = \sqrt{1+81} = \sqrt{82}$ units

29. Radius of cylinder (r) = Radius of each hemisphere (r) = 2.8 cm



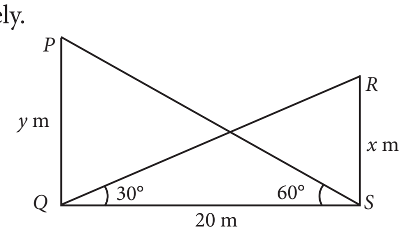
Height of cylinder (h) = 15 cm
Volume of the wood in the toy
= Volume of cylinder
– 2 (Volume of each hemisphere)

$$= \pi r^2 h - 2\left(\frac{2}{3}\pi r^3\right) = \pi r^2\left(h - \frac{4}{3}r\right)$$

$$= \frac{22}{7} \times 2.8 \times 2.8\left(15 - \frac{4}{3}(2.8)\right) = \frac{172.48}{7}\left(\frac{45-11.2}{3}\right)$$

$$= \frac{172.48\,(33.8)}{21} = 277.61\,\text{cm}^3$$

30. Let PQ and RS be two poles of height y m and x m respectively.



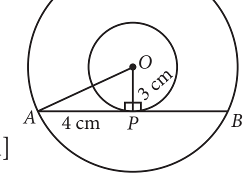
In ΔPQS, $\tan 60° = \frac{y}{20} \Rightarrow y = 20\sqrt{3}$

In ΔRSQ, $\tan 30° = \frac{x}{20} \Rightarrow x = \frac{20}{\sqrt{3}}$

$$\Rightarrow y - x = 20\sqrt{3} - \frac{20}{\sqrt{3}} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.09$$

$\therefore$ Difference between heights of pole is 23.09 m.

31. $OP \perp AB$ [$\because$ Tangent is perpendicular to the radius through the point of contact]



$\therefore$ $AP = BP$ [$\because$ AB is chord to larger circle and $OP \perp AB$]

$\therefore$ $AP = \frac{8}{2} = 4$ cm [$\because$ $AB = 8$ cm]

$OP = \frac{6}{2} = 3$ cm [$\because$ Diameter of smaller circle = 6 cm]

In right ΔOAP, $OA^2 = OP^2 + AP^2$

$= 3^2 + 4^2 = 9 + 16 = 25 \Rightarrow OA = 5$ cm

Thus, diameter of the larger circle is 10 cm.

32. It is given that, $\sqrt{3}$ is an irrational number.

We have to prove that $5 - 2\sqrt{3}$ is an irrational number.

Let us assume $5 - 2\sqrt{3}$ be a rational number.

Then, $5 - 2\sqrt{3} = \frac{a}{b}$, where $b \neq 0$ and $a, b \in R$

$\Rightarrow$ $2\sqrt{3} = 5 - \frac{a}{b}$

$\Rightarrow$ $\sqrt{3} = \left(\frac{5}{2} - \frac{a}{2b}\right)$, which is a rational number.

$\Rightarrow$ $\sqrt{3}$ is a rational number.

But $\sqrt{3}$ is an irrational number. So, our assumption is wrong.

Hence, $5 - 2\sqrt{3}$ is an irrational number.

33. Compare the given equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

(i) For $3x + y - 14 = 0$, $2x + 5y - 5 = 0$,
we have $a_1 = 3, b_1 = 1, c_1 = -14$ and $a_2 = 2, b_2 = 5, c_2 = -5$

Now, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{5}$ and $\frac{c_1}{c_2} = \frac{-14}{-5} = \frac{14}{5}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the lines intersect at a point.

(ii) For $5x + 3y - 15 = 0$, $10x + 6y - 30 = 0$
we have $a_1 = 5, b_1 = 3, c_1 = -15$ and $a_2 = 10, b_2 = 6, c_2 = -30$

Now, $\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-15}{-30} = \frac{1}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, therefore the lines are coincident.

(iii) For $5x - y + 7 = 0$, $15x - 3y + 17 = 0$,
we have $a_1 = 5, b_1 = -1, c_1 = 7$ and $a_2 = 15, b_2 = -3, c_2 = 17$

Now, $\frac{a_1}{a_2} = \frac{5}{15} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{7}{17}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the lines are parallel.

34. (a) Let the missing frequency be f and $h = 3$.
Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$u_i = \frac{x_i - 47.5}{h}$	f_iu_i
40-43	31	41.5	–2	–62
43-46	58	44.5	–1	–58
46-49	60	47.5 = a (let)	0	0
49-52	f	50.5	1	f
52-55	27	53.5	2	54
Total	$\Sigma f_i = 176 + f$			$\Sigma f_i u_i = f - 66$

$\therefore$ Mean $= a + h\left\{\frac{\Sigma f_iu_i}{\Sigma f_i}\right\}$

$\Rightarrow$ $47.2 = 47.5 + 3 \times \left\{\frac{f-66}{176+f}\right\}$

($\because$ Given, mean = 47.2)

$\Rightarrow$ $-0.3 = 3 \times \left\{\frac{f-66}{176+f}\right\} \Rightarrow \frac{-1}{10} = \frac{f-66}{176+f}$

$\Rightarrow$ $-176 - f = 10f - 660 \Rightarrow 11f = 484 \Rightarrow f = 44$

Hence, the missing frequency is 44.

OR

(b) Here, the given frequency distribution is not continuous. So, first we make it continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit.

Class-interval	Frequency
–0.5-9.5	12
9.5-19.5	15
19.5-29.5	21
29.5-39.5	17
39.5-49.5	19
49.5-59.5	6

Here, the highest frequency is 21, which lies in the interval 19.5 – 29.5.

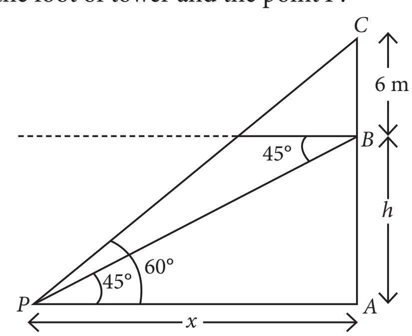
$\therefore$ $l = 19.5, f_1 = 21, f_0 = 15, f_2 = 17$ and $h = 10$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 19.5 + \left(\frac{21-15}{2\times 21 - 15 - 17}\right) \times 10 = 19.5 + \frac{6}{10} \times 10$

$= 19.5 + 6 = 25.5$

35. (a) Let h be the height of tower and x be the distance between the foot of tower and the point P.



Now, ln ΔAPB, we have

$\tan 45° = \frac{AB}{AP} \Rightarrow 1 = \frac{h}{x} \Rightarrow h = x$ ...(i)

In ΔACP, we have

$\tan 60° = \frac{AC}{AP}$

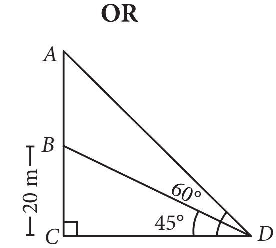
$\Rightarrow \sqrt{3} = \frac{h+6}{x} \Rightarrow \sqrt{3}h = h + 6$ (Using (i))

$\Rightarrow h(\sqrt{3}-1) = 6 \Rightarrow h(1.73 - 1) = 6 \Rightarrow h = \frac{6}{0.73} = 8.22$ m

So, height of the tower = 8.22 m
And, the distance between P and A = 8.22 m

OR

(b)



Let BC be 20 m high building and A is the top of transmission tower AB.

$\therefore$ In $\Delta BCD, \dfrac{BC}{CD} = \tan 45° \Rightarrow \dfrac{BC}{CD} = 1$

$\Rightarrow$ $BC = CD = 20$ m [$\because$ $BC = 20$ cm]

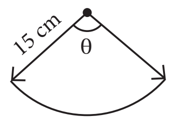
Now, In ΔACD, $\dfrac{AC}{CD} = \tan 60°$

$\Rightarrow \dfrac{AB+BC}{CD} = \tan 60° \Rightarrow \dfrac{AB+20}{20} = \sqrt{3}$

$\Rightarrow AB + 20 = 20\sqrt{3} \Rightarrow AB = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)$

Height of the tower $= 20(\sqrt{3}-1)$ m

36. **(i)** We have, $r = 15$ cm



$l = \dfrac{1}{2}(44) = 22$ cm

We known that $l = 2\pi r\left(\dfrac{\theta}{360°}\right)$

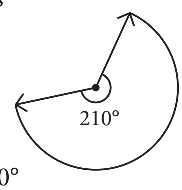
$\Rightarrow \theta = \dfrac{22 \times 360°}{2 \times \dfrac{22}{7} \times 15} = \dfrac{180° \times 7}{15} = 12° \times 7 = 84°$

(ii) Angle made by hour hand in 12 hours = 360°

Angle made by hour hand in 30 minutes $= \left(\dfrac{360°}{12} \times \dfrac{1}{2}\right) = 15°$

(iii) (a) Angle made by minute hand in 60 minutes = 360°
Angle made by minute hand in 35 minutes

$= \dfrac{360°}{60} \times 35 = 210°$



Length of minute hand = 10 cm

$\therefore$ Area swept by minute hand in 35 minutes = Area of sector of an angle 210°

$= \pi r^2 \left(\dfrac{210°}{360°}\right) = \dfrac{22}{7} \times 10 \times 10 \times \dfrac{7}{12} = \dfrac{2200}{12} = 183.33 \text{ cm}^2$

OR

(iii) (b) Angle made by hour hand in 1 hour $= \dfrac{360°}{12} = 30°$
Also, $r = 7$ cm

$\therefore$ Area swept by hour hand in 1 hour = Area of sector of an angle 30°

$= \pi r^2 \times \left(\dfrac{30°}{360°}\right) = \dfrac{22}{7} \times 7 \times 7 \times \dfrac{1}{12} = \dfrac{154}{12} = 12.83 \text{ cm}^2$

Number of hours from 9 a.m. to 5 p.m. = 8
Area swept by hour hand in 1 hour = 12.83 cm^2
$\therefore$ Area swept by hour hand in 8 hours = 12.83 × 8
= 102.64 cm^2

37. Number of cakes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ...
So, it forms an A.P. with first term $a = 3$, $d = 5 - 3 = 2$

(i) Cakes in 17th row $= t_{17} = 3 + 16(2) = 35$
cakes in 10th row $= t_{10} = 3 + 9(2) = 21$

$\therefore$ Required difference = 35 – 21 = 14

(ii) Let n be the number of rows required.

$\therefore S_n = 120$

$\Rightarrow \dfrac{n}{2}[2(3) + (n-1)2] = 120$

$\Rightarrow n^2 + 2n - 120 = 0 \Rightarrow n^2 + 12n - 10n - 120 = 0$

$\Rightarrow (n + 12)(n - 10) = 0 \Rightarrow n = 10$

So, 10 rows are required to put 120 cakes.

(iii) (a) No. of cakes in 5th row $= t_5 = 3 + 4(2) = 11$
No. of cakes in 8th row $= t_8 = 3 + 7(2) = 17$
$\therefore$ Required sum = 11 + 17 = 28
No. of cakes in 30th row $= t_{30} = 3 + 29(2) = 61$

OR

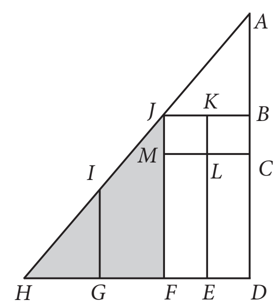
(iii) (b) Here $n = 15$

$\therefore$ $t_{15} = 3 + 14(2) = 3 + 28 = 31$

38. **(i)** In ΔABJ and ΔADH, $\angle B = \angle D = 90°$

$\angle A = \angle A$ (common)

$\therefore$ By AA similarity criterion, $\Delta ABJ \sim \Delta ADH$.



(ii) In ΔADH and ΔJFH, $\angle D = \angle F = 90°$

$\angle H = \angle H$ (common)

$\therefore$ By AA similarity criteria, $\Delta ADH \sim \Delta JFH$

$\therefore \dfrac{AD}{JF} = \dfrac{AH}{JH} \Rightarrow \dfrac{12}{JM + MF} = \dfrac{8}{4} \Rightarrow \dfrac{12 \times 4}{8} = JM + 4$

$\Rightarrow 6 = JM + 4 \Rightarrow JM = 2$ m

(iii) (a) In ΔADH, $IG \parallel AD$

$\therefore \dfrac{IH}{AI} = \dfrac{HG}{GD}$ (By Thales theorem)

$\Rightarrow \dfrac{y+3}{2y} = \dfrac{y}{2y-1}$

$\Rightarrow (y + 3)(2y - 1) = y(2y) \Rightarrow 2y^2 - y + 6y - 3 = 2y^2$

$\Rightarrow 5y - 3 = 0 \Rightarrow y = 3/5$

OR

(iii) (b) Since, $\Delta ABJ \sim \Delta ADH$ [By AA similarity criterion]

$\therefore \dfrac{AB}{AD} = \dfrac{AJ}{AH} \Rightarrow \dfrac{AB}{AJ} = \dfrac{5}{7}$

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